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Resonance phenomena in cylindrical shell with a spherical inclusion in the presence of an internal compressible liquid and an external elastic medium

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Abstract

In the present paper a method is proposed to investigate the behaviour of the axisymmetric system consisting of an infinite thin elastic cylindrical shell submerged in an unbounded elastic medium, filled with an ideal compressible liquid and containing a vibrating spherical inclusion, under periodic dynamic action. The goal is the analysis of the so-called "resonance" phenomena; namely: finding conditions for their appearance, and possible control by means of characteristic parameters of the hydroelastic system under consideration. The technique presented in this work was developed during the realization of a project on elaboration of methods of renewal of oil production in foul wells at the Theory of Vibration Department of the S.P. Timoshenko Institute of Mechanics of the Ukrainian Academy of Science. This mathematical technique allows rewriting the general solution of the corresponding mathematical physics equations from one coordinate system to another, so as to get an exact analytical solution (as a Fourier series) of the interaction problem for a collection of rigid and elastic bodies.

Keywords: Vibrations; Helmholtz equation; Thin elastic cylindrical shell; Elastic medium; Ideal compressible liquid; Spherical inclusion

1. Introduction

The problem of fluid–structure interaction is wide and covers many types of both fluid and structural behaviour. Such problems can be interesting for researching processes of vibrodisplacement and localization, decontamination of liquid medium, airing and dispersion; in bioacoustics and cardiovascular medicine (for instance, for some problems involving blood flow, where fluid and structure models are coupled); in nondestructive testing (for instance, the scattering of acoustic waves can give important information about the internal composition of solids and fluids, yielding information about internal inhomogeneities, asymmetries and defects from the scattering pattern); in technologies of resumption of oil production in foul wells, etc. For many years, numerous authors have been interested in the dynamics of fluid–structure interaction both for unbounded domains of either fluid or structure and for delimited ones, involving both motionless and flowing fluids.

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List of Symbols

γ_m	density of the shell material
γ_s	density of the external elastic medium
λ, μ	Lamé constants for the external elastic medium
v	Poisson coefficient of the shell material
v_s	Poisson coefficient of the elastic medium
ρ_0	the shell radius
$\sigma_{\rho\rho}$, $\sigma_{\rho z}$, $\sigma_{\rho\varphi}$	strains in the elastic medium
ω	frequency vibrations of the sphere
γ_1	density of the liquid inside the shell
$\Phi^{(1)}$	wave potential inside the shell
$\Phi^{(2)}$, ψ, χ	displacement potentials outside the shell
a_l, a_t	speeds of longitudinal and transversal waves in the elastic medium
$A(\xi)$, $B^{(1)}(\xi)$, $B^{(2)}(\xi)$, $D(\xi)$	unknown functions
c^*	c_1/c_m
c_1	sound speed in the liquid inside the shell
E	Young's modulus of the shell material
E_s	Young's modulus of the elastic medium
h	the shell thickness
O,ρ,z,φ	the cylindrical coordinate system
O,r,θ,φ	the spherical coordinate system
$p^{(1)}$	hydrodynamic pressure inside the shell
r_0	radius of the spherical inclusion
t	time
w	the shell deflection
u	displacement of points of the shell middle surface in axial direction
$U^{(1)}$	speed of liquid motion
$\vec{U}_s = \{U_z, U_\rho, U_\varphi\}$	vector of displacements of the elastic medium
x_n	unknown constants

As thin shells are used in a variety of applications, they continue to arouse the interest of researchers who study their behaviour under dynamic and static loads. Thin shells are commonly used in the aeronautical industry, in the generation of nuclear energy and in the construction industry. Cylindrical shapes are also widely used in various forms as pressurized containers, pipes, and structural components. The presence of a liquid inside a shell (usually, these shells are made as thin as possible for weight and cost considerations), as has been already mentioned, has an important influence on the dynamic behaviour of the structure and can create problems which are difficult to solve.

The free vibration characteristics of a fluid-surrounded or fluid-filled cylindrical shell subjected to various loads have been of great concern in engineering design. Hence, many investigations in this area have been carried out. The free vibration analysis of two infinitely long, coaxial cylinders containing fluid was performed by Krajcinovic (1974). Chen and Rosenberg (1975) derived a frequency equation for two concentrically arranged circular cylindrical shells containing and separated by incompressible fluid and obtained an approximate closed-form solution. The free vibration of an infinitely long cylindrical shell under axisymmetric hydrodynamic pressures of the external and internal fluids was studied using a Fourier cosine transformation by Endo and Tosaka (1989). Tani et al. (1989) performed a study on the free vibration of clamped coaxial cylindrical shells partially filled with incompressible inviscid fluid; the theoretical analysis was based on the Galerkin method and the velocity potential theory for the fluid.

A lot of papers were devoted to the multi-linked problems of interaction between shells and medium. Systematic reviews of these works have been presented in the monographs by Vol'mir (1979), Guz' et al. (1978), Shenderov (1972), in articles by Olsson (1993, 1990), Iakovlev (2004), Scandrett and Canright (1991), and in other publications.

It should be mentioned that simplicity of boundary surfaces was the characteristic feature of the overwhelming majority of the problems considered. As for hydroelasticity problems, interaction of the single shells (or rigid bodies) with the surrounding medium [for example, in the article by Iakovlev (2002), or in the monograph by Shenderov (1972)], or interaction of the bodies with the same type of surfaces—families of parallel cylinders [for example, in the monograph by Shenderov (1972) and articles by Doolittle and Uberall (1966), Magrab (1972), Honarvar and Sinclair (1996)] or families of spherical (spheroid) bodies [for example, in the work by Scandrett and Canright (1991)]—in the ideal compressible and incompressible liquids and in a viscous medium [in the monograph by Guz' (1998)] were also considered.

For the last 10 years the work on dynamics of interaction of shells with inclusions (modelled as interaction of bodies with different geometric forms) have appeared [for example, articles by Kubenko and Dzyuba (2000, 2001), Kubenko and Kruk (1999) and Olsson (1990, 1993)]. For such problems there are many approaches (both analytical and numerical) for their solution. Among them are the ≪T-matrix method≫, suggested by Waterman (1969), combining separation of variables in the Helmholtz equation and solving the integral equation; ≪null-field method≫, investigated in detail by Bates and Wall (1977), based on the same idea; the method of series together with using the addition theorems for the corresponding wave functions, which enables the boundary problems to be reduced to investigating and solving infinite systems of algebraic equations [the monograph by Guz' et al. (1978), for example, illustrates this approach]. Such methods can be applied for solving problems on sound diffraction on surfaces of arbitrary form, on sound diffraction on elastic bodies and on the problems of elastic wave diffraction in a solid, on diffraction of waves in an elastic medium, etc.

When fluid–structure interaction problems cannot be solved analytically, numerical methods are applied. Two outstanding numerical methods are the boundary element method (BEM) and the finite element method (FEM). With numerical methods, special features have to be introduced in order to be able to deal with fluid–structure interaction problems in unbounded domains. The BEM is a boundary discretization method and hence presents an efficient tool for solving radiation problems in unbounded domains. However, the method fails at so-called "critical frequencies", as weak singular boundary integrals occur. This disadvantage does not appear when the finite element method is used; but, as the FEM is a domain discretization method, it is actually not appropriate for solving problems in infinite domains. To overcome this difficulty, the theory of semi-infinite and infinite elements has been developed; the so-called "Dirichlet-to-Neumann" boundary condition for solving problems in infinite domains has been elaborated. Another approach is to introduce a boundary condition on the outer boundary of the computational domain. This boundary condition has to simulate the infinite outer domain: i.e., it represents the influence of the unbounded domain on the finite domain discretized with finite elements. To overcome the inability to deal with open field scattering problems, the FEM has also been coupled, for example, with the bimoment method, modal expansion and absorbing boundary condition. But a full literature review regarding this subject is not the main goal of the present paper. So, as examples, the articles by Berot and Peseux (1998), Kochupillai et al. (2002), Mallardo and Aliabadi (1998), Selmane and Lakis (1997), Zhang et al. (2002) can be mentioned as works in which the dynamics of the interaction between shells and liquid medium using the FEM/BEM technique is considered.

Axisymmetric systems, consisting of an infinitely long thin elastic cylindrical shell, submerged in an unbounded elastic medium, filled by an ideal compressible liquid and containing a vibrating spherical inclusion, are considered in the present paper. The analysis of phenomena in these systems is mainly based on the papers by Kubenko and Dzyuba (2000, 2001, 2003).

The potential function, which defines pulsating motion of a spherical inclusion in compressible (approximately acoustic) liquid, filling a circular cylindrical cavity, was constructed in papers by Kubenko and Dzyuba (2000, 2001) and Kubenko and Kruk (1999). In these papers it is supposed that a source of spherical form is situated on the cavity axis, thus there is axial symmetry. In the paper by Kubenko and Dzyuba (2000) the authors considered an axisymmetric problem on vibrations of a sphere in a rigid cylindrical cavity containing compressible liquid. A problem of interaction of a stiff cylindrical cavity with a finite number of spherical inclusions in compressible liquid filling the cavity was described in the paper by Kubenko and Dzyuba (2003). A solution of the problem of construction of the potential function, defined by motion of a spherical body in a predetermined manner in compressible liquid, filling a thin elastic cylindrical shell, was obtained by Kubenko and Dzyuba (2001); numerical results of concrete problems were given.

The dynamic behaviour of the hydroelastic systems under consideration in the present paper can be described by equations modelling movement of an ideal compressible liquid, of a thin elastic shell and an elastic medium. The equations of acoustic theory, of the theory of thin elastic shells based on the Kirchhoff–Love hypotheses and the Lamé equations have been considered.

The main difficulty in the problem under consideration consists in so-called triple interaction: between the shell and the fluid and the elastic medium from one side, and between bodies with different geometries (the cylindrical shell and the spherical source) from the other. It is difficult to get a total analytical solution which takes into account both all possible waves many times scattered on the bodies of the system considered and the important influence of the fluid and the elastic medium on the system dynamic behaviour, because separate components of the total solution belong to different coordinate systems.

Overcoming this difficulty has required elaboration of the mathematical technique allowing rewriting a total solution of corresponding equations of mathematical physics from one coordinate system to another. Thus, solution of the problem under consideration has been based on the possibility of representing particular solutions of the Helmholtz equation, written through the cylindrical coordinates, with the help of particular solutions of the same equation written through the spherical coordinates, and vice versa. Due to this mathematical procedure one has total solutions in the coordinate system of each body of the system under consideration which enable to satisfy the boundary conditions. As a result of satisfying the boundary conditions, an infinite system of algebraic equations has been obtained to define coefficients of expansion of the fluid velocity potential into the Fourier series according to Legendre polynomials. The wave potential has been chosen as a desired function, through which all the rest of the characteristics of the system under consideration can be expressed.

Two mechanical systems have been considered (depending on the character of the distribution of the vibrating velocity along the surface of the spherical inclusion): with a pulsating sphere (the case of uniform amplitude-phase distribution of the velocity along the surface) and with the sphere oscillating along the shell axis. The hydrodynamic characteristics of the liquid inside the cylindrical volume have been investigated versus the frequency in order to analyse "resonance" phenomena. The comparison with frequency dependencies of the hydrodynamic characteristics of the corresponding plane axisymmetric hydroelastic system has been made.

2. Problem statement

A thin infinitely long elastic circular cylindrical shell with radius ρ_0 is filled with an ideal compressible liquid, the properties of which are described by the speed of sound c_1 and the density γ_1. The shell is enclosed by an unbounded elastic medium with density γ_s. A spherical source of radius r_0 is located inside the shell on its axis. The source surface vibrates harmonically according to a predetermined time law: $\mathrm{e}^{-\mathrm{i}\omega t}$. The spherical body and the cylindrical shell do not have points of contact. It is necessary to determine an internal and an external (with respect to the shell) wave field, and to evaluate their interaction and dependency versus the excitation frequency.

Let us refer the cylindrical shell to the cylindrical coordinate system (O, ρ, z, φ), in which the Oz-axis coincides with the cylinder axis. Let us connect the spherical coordinates r, θ, φ with the spherical body centre lying on the cylinder axis (Fig. 1)·

Liquid motion is supposed to be irrotational, axisymmetric in view of the symmetry of the problem geometry, and amplitudes of the disturbances in the fluid are assumed to be small. Therefore, a boundary problem for the internal medium consists in searching for a solution of the Helmholtz equation:

$$\left(\nabla^2 - \frac{1}{c_1^2}\frac{\partial^2}{\partial t^2}\right)\Phi^{(1)}\,\mathrm{e}^{-\mathrm{i}\omega t} = 0. \tag{1}$$

Here, ∇^2 is the Laplacian, $\Phi^{(1)}$ is the wave potential inside the shell, ω is circular frequency of vibrations of the sphere which is known, and $\mathrm{i}^2 = -1$.

It should be noted that time multiplier $\mathrm{e}^{-\mathrm{i}\omega t}$ should be kept in mind henceforth and all system characteristics will be considered as independent of the angle coordinate φ because of the axial symmetry of the problem.

The desired solution must satisfy the following boundary conditions:

(i) on the surface of contact of the shell and liquid filler (nonpenetration condition):

$$\left.\frac{\partial\Phi^{(1)}(\rho,z)}{\partial\rho}\right|_{\rho=\rho_0} = \mathrm{i}\omega w(z). \tag{2}$$

(ii) on the sphere surface

$$\left.\frac{\partial\Phi^{(1)}(r,\theta)}{\partial r}\right|_{r=r_0} = V(\theta). \tag{3}$$

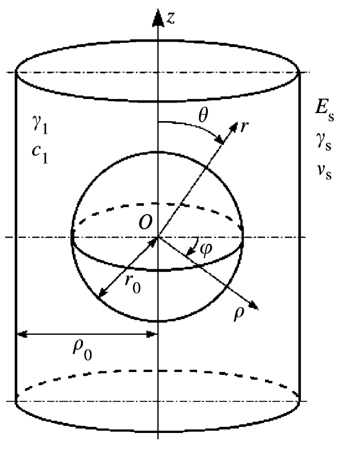

Fig. 1. Geometry of the hydroelastic system considered.

Also it must satisfy the condition of attenuation of the disturbances at infinity

$$\lim_{r\to\infty} \Phi^{(1)} = 0 \tag{4}$$

or the condition of boundedness of the disturbances inside the cylindrical volume

$$\lim_{\rho\to 0} \Phi^{(1)} = \text{const.} \tag{5}$$

In Eqs. (2)–(5), w is the shell deflection (w is considered to be positive inward); $V(\theta)$ is a known function, which can be represented as the series according to the Legendre polynomials:

$$V(\theta) = \sum_{n=0}^{\infty} V_n P_n(\cos\theta).$$

Let us recall that the wave potential $\Phi^{(1)}$ is connected with the pressure and the velocity of liquid motion by the formulas

$$p^{(1)} = \mathrm{i}\gamma_1\omega\Phi^{(1)}, \ \vec{\mathbf{U}}^{(1)} = \operatorname{grad}\Phi^{(1)}. \tag{6}$$

The behaviour of the external elastic medium can be described by the vector Lamé equation written for the case of steady motion with exponential time dependency:

$$a_l^2 \operatorname{grad}\operatorname{div}\vec{\mathbf{U}}_s - a_t^2 \operatorname{rot}\operatorname{rot}\vec{\mathbf{U}}_s + \omega^2\vec{\mathbf{U}}_s = 0,$$
$$a_l^2 = \frac{\lambda+2\mu}{\gamma_s}, \ a_t^2 = \frac{\mu}{\gamma_s}. \tag{7}$$

Here, $\vec{\mathbf{U}}_s = \{U_z, U_\rho, U_\varphi\}$ is a vector of the displacements of the elastic medium; λ and μ are Lamé's constants, connected with the material constants with the help of the following relations:

$$\lambda = \frac{E_s v_s}{(1+v_s)(1-2v_s)}, \ \mu = \frac{E_s}{1+v_s}.$$

The subscript s (solid) points out that this definition is related to the elastic medium; v_s is the Poisson's ratio of the elastic medium and E_s the modulus of elasticity.

In general, for the elastic medium, in contrast to the liquid one, the vector of the displacements $\vec{\mathbf{U}}_s$ cannot be simply expressed through a scalar function of the potential. In addition it is necessary to consider a vector function (Guz' et al., 1978; Morse and Feshbach, 1960). Thus, the vector of the displacements of elastic medium $\vec{\mathbf{U}}_s$ should be presented as follows:

$$\vec{\mathbf{U}}_s = \vec{\mathbf{U}}_s^{(G)} + \vec{\mathbf{U}}_s^{(M)} + \vec{\mathbf{U}}_s^{(N)}, \tag{8}$$

where

$$\vec{\mathbf{U}}_s^{(G)} = \operatorname{grad}\Phi^{(2)} = \vec{\mathbf{e}}_z\frac{\partial\Phi^{(2)}}{\partial z} + \vec{\mathbf{e}}_\rho\frac{\partial\Phi^{(2)}}{\partial\rho} + \vec{\mathbf{e}}_\varphi\frac{1}{\rho}\frac{\partial\Phi^{(2)}}{\partial\varphi},$$
$$\vec{\mathbf{U}}_s^{(M)} = \operatorname{rot}(\vec{\mathbf{e}}_z\psi) = \vec{\mathbf{e}}_\varphi\frac{1}{\rho}\frac{\partial\psi}{\partial\varphi} - \vec{\mathbf{e}}_\rho\frac{\partial\psi}{\partial\rho},$$
$$\vec{\mathbf{U}}_s^{(N)} = \frac{a_t}{\omega}\operatorname{rot}\operatorname{rot}(\vec{\mathbf{e}}_z\chi) = \frac{a_t}{\omega}\left[\vec{\mathbf{e}}_z\left(\frac{\omega^2}{a_t^2}\chi + \frac{\partial^2\chi}{\partial z^2}\right) + \vec{\mathbf{e}}_\rho\frac{\partial^2\chi}{\partial z\,\partial\rho} + \vec{\mathbf{e}}_\varphi\frac{1}{\rho}\frac{\partial^2\chi}{\partial z\,\partial\varphi}\right].$$

Here $\Phi^{(2)}$, ψ, χ are solutions of the following scalar wave equations:

$$\nabla^2\Phi^{(2)} + \frac{\omega^2}{a_l^2}\Phi^{(2)} = 0; \quad \nabla^2\psi + \frac{\omega^2}{a_t^2}\psi = 0; \quad \nabla^2\chi + \frac{\omega^2}{a_t^2}\chi = 0. \tag{9}$$

Components of the displacement vector and the stresses in the elastic medium are presented in Appendix A.

Let us note that, for an axisymmetric case, deformations of the elastic medium do not depend on the angular coordinate φ of the coordinate systems considered. Thus, the displacement component U_φ vanishes, and consequently the scalar function ψ also vanishes: $\psi = 0$ (Guz' et al., 1978; Morse and Feshbach, 1960).

The statement of the boundary value problem for the external medium will be completed by accounting for certain boundary conditions on the surface of contact of the shell and the external medium and the radiation condition at infinity for all solutions of Eq. (9). Conditions at the "shell-external medium" contact surface are expressed by

(i) the equality of normal displacements of the elastic medium and the shell:

$$U_z(\rho, z)\big|_{\rho=\rho_0} = w(z). \tag{10}$$

(ii) the equality of axial displacements of the elastic medium and the shell:

$$U_z(\rho, z)\big|_{\rho=\rho_0} = u(z). \tag{11}$$

Henceforth the dimensionless variables introduced are used as follows:

$$\bar{r} = \frac{r}{\rho_0}, \bar{f} = \frac{\gamma_1}{\gamma_m}, \bar{t} = \frac{tc_1}{\rho_0}, \bar{\omega} = \frac{\omega\rho_0}{c_1}, \bar{U} = \frac{U}{c_1}, \bar{\Phi}^{(1)} = \frac{\Phi^{(1)}}{\rho_0 c_1},$$
$$\bar{\psi} = \frac{\psi}{\rho_0^2}, \bar{p} = \frac{p}{\gamma_1 c_1^2}. \tag{12}$$

Hereafter the overbar is omitted in all expressions.

The cylindrical shell is subjected to the internal fluid load and the external elastic medium load. Such loading is symmetric relative to the Oz-axis. Consequently, deformations of the shell median surface will not depend on the angle of rotation around the Oz-axis (i.e., on the angular coordinate φ), and the displacements of points of the median surface along the arc will be identically equal to zero.

Taking into consideration the linear theory of shells (Vol'mir, 1979), based on the Kirchhoff–Love hypotheses, and assuming that the deflections are small compared with the shell thickness, let us write out the following differential equations of shell motion for the case of axisymmetric deformation:

$$\frac{\partial^2 u(z)}{\partial z^2} - v\frac{\partial w(z)}{\partial z} = -\omega^2 c^{*^2} u(z),$$
$$-v\frac{\partial u(z)}{\partial z} + \left(1 + \frac{h^2}{12}\frac{\partial^4}{\partial z^4}\right) w(z) = \left(\frac{f}{h} q(1, z) + \omega^2 w(z)\right) c^{*^2}, \tag{13}$$

where $c^* = c_1/c_m$. The sound speed in the shell material is $c_m = \sqrt{E/[\gamma_m(1 - v^2)]}$. It is recalled that hereafter only the dimensionless variables (12) are used.

In the above: u is the displacement of points of the shell median surface in the axial direction; γ_m is the density of the cylindrical shell material; E is the modulus of elasticity; v is the Poisson ratio; q is the distributed load on the shell both from the external and internal side defined as follows:

$$q\big|_{\rho=\rho_0} = (-p_1 + \sigma_{\rho\rho})\big|_{\rho=\rho_0} = (-\mathrm{i}\omega\Phi^{(1)} + \sigma_{\rho\rho})\big|_{\rho=\rho_0}. \tag{14}$$

3. Construction of solution of the boundary value problem

The field inside the shell is a result of the superposition of waves scattered many times on the surfaces of the hydroelastic system considered. The liquid velocity potential inside the cylindrical volume $\Phi^{(1)}$ has already been chosen as a desired function. Let it be the sum of two functions: one function expresses waves scattered by the spherical inclusion (the solution of Eq. (1) in spherical coordinates), and another function describes disturbances in the fluid arising from scattering on the shell walls (the solution of Eq. (1) in cylindrical coordinates):

$$\Phi^{(1)} = \Phi^{(1)}_{\mathrm{sph}} + \Phi^{(1)}_{\mathrm{cyl}}.$$

The solution $\Phi^{(1)}_{\mathrm{sph}}$, damped as $r \to \infty$, has the form

$$\Phi^{(1)}_{\mathrm{sph}}(r, \theta) = \sum_{n=0}^{\infty} x_n h_n(r\omega) P_n(\cos\theta), \tag{15}$$

where P_n are the Legendre polynomials; x_n are undetermined constants; h_n are the spherical Hankel functions of the first kind.

The particular solution of the Helmholtz Eq. (1) in cylindrical coordinates has the form

$$\Phi^{(1)}_{\text{cyl}}(\rho, z, \xi) = B^{(1)}(\xi)\mathrm{J}_0\left(\sqrt{\omega^2 - \xi^2}\rho\right)\mathrm{e}^{\mathrm{i}\xi z}.$$

This solution determines cylindrical waves with the wavenumber ξ running in the positive direction of the Oz-axis. Combining all possible values of ξ, one can write the field $\Phi^{(1)}_{\text{cyl}}$, which is finite everywhere inside the shell, according to condition (5) as follows:

$$\Phi^{(1)}_{\text{cyl}}(\rho, z) = \int_{-\infty}^{\infty} B^{(1)}(\xi)\mathrm{J}_0\left(\sqrt{\omega^2 - \xi^2}\rho\right)\mathrm{e}^{\mathrm{i}\xi z}\mathrm{d}\xi. \tag{16}$$

Here J_0, the Bessel function of zero order.

With the help of the correlations which link the cylindrical wave functions to the spherical ones and vice versa,

$$h_n(\omega r)P_n(\cos\theta) = \frac{\mathrm{i}^{-n}}{2\omega}\int_{-\infty}^{\infty} P_n\left(\frac{\xi}{\omega}\right)\mathrm{H}_0\left(\sqrt{\omega^2 - \xi^2}\rho\right)\mathrm{e}^{\mathrm{i}\xi z}\,\mathrm{d}\xi,$$
$$\mathrm{e}^{\mathrm{i}\xi z}\mathrm{J}_0\left(\sqrt{\omega^2 - \xi^2}\rho\right) = \sum_{n=0}^{\infty}\mathrm{i}^n(2n+1)P_n\left(\frac{\xi}{\omega}\right)\mathrm{j}_n(\omega r)P_n(\cos\theta),$$

the total "internal" solution can be written in the coordinate system of each body as a series with separate variables of the same coordinate system:

$$\Phi^{(1)}(\rho, z) = \int_{-\infty}^{\infty}\left[A(\xi)\mathrm{H}_0\left(\sqrt{\omega^2 - \xi^2}\rho\right) + B^{(1)}(\xi)\mathrm{J}_0\left(\sqrt{\omega^2 - \xi^2}\rho\right)\right]\mathrm{e}^{\mathrm{i}\xi z}\,\mathrm{d}\xi, \tag{17}$$

$$A(\xi) = \frac{1}{2\omega}\sum_{n=0}^{\infty} x_n \mathrm{i}^{-n} P_n\left(\frac{\xi}{\omega}\right),$$

$$\Phi^{(1)}(r, \theta) = \sum_{n=0}^{\infty}[x_n\mathrm{h}_n(\omega r) + B_n\mathrm{j}_n(\omega r)]P_n(\cos\theta), B_n = \mathrm{i}^n(2n+1)\int_{-\infty}^{\infty} B^{(1)}(\xi)P_n\left(\frac{\xi}{\omega}\right)\mathrm{d}\xi. \tag{18}$$

Here j_n is the spherical Bessel function of order n, H_0 the Hankel function of the first kind and zero order and $A(\xi)$, and $B^{(1)}(\xi)$ are undetermined functions.

The external solutions, which are so-called "radiating solutions" satisfying the radiation conditions, have the following forms:

$$\Phi^{(2)}(\rho, z) = \int_{-\infty}^{\infty} B^{(2)}(\xi)\mathrm{H}_0\left(\sqrt{\frac{\omega^2}{a_l^2} - \xi^2}\rho\right)\mathrm{e}^{\mathrm{i}\xi z}\,\mathrm{d}\xi, \tag{19}$$

$$\chi(\rho, z) = \int_{-\infty}^{\infty} D(\xi)\mathrm{H}_0\left(\sqrt{\frac{\omega^2}{a_t^2} - \xi^2}\rho\right)\mathrm{e}^{\mathrm{i}\xi z}\,\mathrm{d}\xi. \tag{20}$$

where $B^{(2)}(\xi)$ and $D(\xi)$ are unknown functions to be determined from the boundary conditions.

4. Satisfying the boundary conditions

First of all let us find mathematical relations between the component $\sigma_{\rho\rho}$ of the stress tensor, the displacements of the elastic medium U_z, U_ρ and the desired functions with the help of well-known correlations (Guz' et al., 1978; Morse and Feshbach, 1960) between displacements and scalar potentials $\Phi^{(2)}$ and χ (Eqs. (A.1)), stresses and displacements (Eqs. (A.2)):

$$U_z = \int_{-\infty}^{\infty}\left[\mathrm{i}\xi B^{(2)}(\xi)\mathrm{H}_0\left(\sqrt{\frac{\omega^2}{a_l^2} - \xi^2}\rho\right) + \frac{a_t}{\omega}\left(\frac{\omega^2}{a_t^2} - \xi^2\right)D(\xi)\mathrm{H}_0\left(\sqrt{\frac{\omega^2}{a_t^2} - \xi^2}\rho\right)\right]\mathrm{e}^{\mathrm{i}\xi z}\,\mathrm{d}\xi, \tag{21}$$

$$U_\rho = -\int_{-\infty}^{\infty}\left[B^{(2)}(\xi)\sqrt{\frac{\omega^2}{a_l^2}-\xi^2}\mathrm{H}_1\left(\sqrt{\frac{\omega^2}{a_l^2}-\xi^2}\rho\right)+\frac{a_t}{\omega}\mathrm{i}\xi\sqrt{\frac{\omega^2}{a_t^2}-\xi^2}D(\xi)\mathrm{H}_1\left(\sqrt{\frac{\omega^2}{a_t^2}-\xi^2}\rho\right)\right]\mathrm{e}^{\mathrm{i}\xi z}\,\mathrm{d}\xi, \tag{22}$$

$$\sigma_{\rho\rho} = \int_{-\infty}^{\infty}\left[B^{(2)}(\xi)b^{(2)}(\xi,\rho)+D(\xi)d(\xi,\rho)\right]\mathrm{e}^{\mathrm{i}\xi z}\,\mathrm{d}\xi, \tag{23}$$

where the symbols introduced here are presented in Appendix B.

It is mathematically convenient to satisfy the boundary condition on the shell surface in the Fourier image space.

Using the Fourier transform according to z-coordinate in the Eqs. (13) and taking into account Eq. (14) leads to the following expressions in the image space, which link the shell deflections and axial displacements with the velocity potential of the "internal" liquid and the stresses in the external elastic medium:

$$w^F(\xi) = R(\xi)[\sigma_{\rho\rho}^F - \mathrm{i}\omega\Phi^{(1)F}(1,\xi)], \tag{24}$$

$$u^F(\xi) = R_u(\xi)[\sigma_{\rho\rho}^F - \mathrm{i}\omega\Phi^{(1)F}(1,\xi)]. \tag{25}$$

Symbols introduced in these formulas are also presented in Appendix B. The superscript F denotes the Fourier transform:

$$w^F(\xi) = \int_{-\infty}^{\infty} w(z)\mathrm{e}^{-\mathrm{i}\xi z}\mathrm{d}\xi.$$

Further it is necessary to rewrite the boundary conditions on the thin elastic cylindrical shell surface (2), (10), (11) into the Fourier image space:

$$\left.\frac{\partial\Phi^{(1)F}(\rho,\xi)}{\partial\rho}\right|_{\rho=\rho_0} = \mathrm{i}\omega w^F(\xi), \tag{26}$$

$$\left.U_\rho^F(\rho,\xi)\right|_{\rho=\rho_0} = w^F(\xi), \tag{27}$$

$$\left.U_z^F(\rho,\xi)\right|_{\rho=\rho_0} = u^F(\xi). \tag{28}$$

Here, in accordance with formulas (17), (21) and (22),

$$\Phi^{(1)F}(\rho,\xi) = A(\xi)\mathrm{H}_0\left(\sqrt{\omega^2-\xi^2}\rho\right)+B^{(1)}(\xi)\mathrm{J}_0\left(\sqrt{\omega^2-\xi^2}\rho\right), \tag{29}$$

$$U_\rho^F(\rho,\xi) = B^{(2)}(\xi)\sqrt{\frac{\omega^2}{a_l^2}-\xi^2}\mathrm{H}_1\left(\sqrt{\frac{\omega^2}{a_l^2}-\xi^2}\rho\right)+\frac{a_t}{\omega}\mathrm{i}\xi\sqrt{\frac{\omega^2}{a_t^2}-\xi^2}D(\xi)\mathrm{H}_1\left(\sqrt{\frac{\omega^2}{a_t^2}-\xi^2}\rho\right), \tag{30}$$

$$U_z^F(\rho,\xi) = \mathrm{i}\xi B^{(2)}(\xi)\mathrm{H}_0\left(\sqrt{\frac{\omega^2}{a_l^2}-\xi^2}\rho\right)+\frac{a_t}{\omega}\left(\frac{\omega^2}{a_t^2}-\xi^2\right)D(\xi)\mathrm{H}_0\left(\sqrt{\frac{\omega^2}{a_t^2}-\xi^2}\rho\right), \tag{31}$$

and the expressions for the functions $w^F(\xi)$, $u^F(\xi)$ are defined by formulas (24) and (25).

Satisfying the boundary conditions (26)–(28) leads to the system of equations regarding the functions $B^{(1)}(\xi)$, $B^{(2)}(\xi)$, and $D(\xi)$. Solutions of this system represent mathematical expressions which link unknown functions with the desired coefficients of the expansion of the liquid velocity potential (which determines the field, scattered by the sphere) in the Fourier series according to the Legendre polynomials:

$$B^{(1)}(\xi) = -\frac{\sqrt{\omega^2-\xi^2}\mathrm{H}_1\left(\sqrt{\omega^2-\xi^2}\right)(1+M(\xi))+\omega^2R(\xi)\mathrm{H}_0\left(\sqrt{\omega^2-\xi^2}\right)}{\sqrt{\omega^2-\xi^2}\mathrm{J}_1\left(\sqrt{\omega^2-\xi^2}\right)(1+M(\xi))+\omega^2R(\xi)\mathrm{J}_0\left(\sqrt{\omega^2-\xi^2}\right)}A(\xi), \tag{32}$$

$$B^{(2)}(\xi) = \frac{\mathrm{H}_0\left(\sqrt{\omega^2-\xi^2}\right)\mathrm{J}_1\left(\sqrt{\omega^2-\xi^2}\right) - \mathrm{J}_0\left(\sqrt{\omega^2-\xi^2}\right)\mathrm{H}_1\left(\sqrt{\omega^2-\xi^2}\right)}{\sqrt{\omega^2-\xi^2}\mathrm{J}_1\left(\sqrt{\omega^2-\xi^2}\right)(1+M(\xi)) + \omega^2 R(\xi)\mathrm{J}_0\left(\sqrt{\omega^2-\xi^2}\right)} \mathrm{i}\omega M^{(1)}(\xi) \times \sqrt{\omega^2-\xi^2}A(\xi), \tag{33}$$

$$D(\xi) = -\frac{\mathrm{H}_0\left(\sqrt{\omega^2-\xi^2}\right)\mathrm{J}_1\left(\sqrt{\omega^2-\xi^2}\right) - \mathrm{J}_0\left(\sqrt{\omega^2-\xi^2}\right)\mathrm{H}_1\left(\sqrt{\omega^2-\xi^2}\right)}{\sqrt{\omega^2-\xi^2}\mathrm{J}_1\left(\sqrt{\omega^2-\xi^2}\right)(1+M(\xi)) + \omega^2 R(\xi)\mathrm{J}_0\left(\sqrt{\omega^2-\xi^2}\right)} \mathrm{i}\omega M^{(2)}(\xi) \times \sqrt{\omega^2-\xi^2}A(\xi); \tag{34}$$

symbols introduced here are presented in Appendix B.

From the condition on the surface of the vibrating sphere (when $r = r_0$), Eq. (3), after taking into consideration the potential in the form (18), one can obtain

$$\sum_{n=0}^{\infty} x_n \mathrm{h}_n'(\omega r_0)\omega P_n(\cos\theta) + \sum_{n=0}^{\infty} B_n \mathrm{j}_n'(\omega r_0)\omega P_n(\cos\theta) = \sum_{n=0}^{\infty} V_n P_n(\cos\theta),$$

where the prime designates a derivative with respect to the argument of a spherical function.

It follows that for each n the following equality arises (by virtue of the orthogonality of the Legendre polynomials)

$$x_n \mathrm{h}_n'(\omega r_0)\omega + B_n \mathrm{j}_n'(\omega r_0)\omega = V_n. \tag{35}$$

Eqs. (18), (32), (35) lead to an infinite system of algebraic equations for the determination of the desired constants x_n:

$$x_n - \frac{1}{2\omega}\frac{\mathrm{j}_n'(\omega r_0)}{\mathrm{h}_n'(\omega r_0)}(2n+1)\sum_{m=0}^{\infty} i^{n-m} q_{mn} x_m = \frac{V_n}{\omega \mathrm{h}_n'(\omega r_0)}, n = 0, 1, 2\ldots, \tag{36}$$

where coefficients q_{mn} have the form

$$q_{mn} = 2\int_0^{\infty} \frac{\sqrt{\omega^2-\xi^2}\mathrm{H}_1\left(\sqrt{\omega^2-\xi^2}\right)(1+M(\xi)) + \omega^2 R(\xi)\mathrm{H}_0\left(\sqrt{\omega^2-\xi^2}\right)}{\sqrt{\omega^2-\xi^2}\mathrm{J}_1\left(\sqrt{\omega^2-\xi^2}\right)(1+M(\xi)) + \omega^2 R(\xi)\mathrm{J}_0\left(\sqrt{\omega^2-\xi^2}\right)} P_n\left(\frac{\xi}{\omega}\right) P_m\left(\frac{\xi}{\omega}\right) \mathrm{d}\xi, \tag{37}$$

when the sum of indices $n+m$ is even; otherwise $q_{mn} = 0$.

The infinite system of algebraic Eq. (36) belongs to the class of normal type systems. It had been demonstrated by Kubenko and Dzyuba (2000) that the determinant of a system similar to system (36) is the determinant of the normal type. Thus, the system obtained has a unique bounded solution as a system of normal type. This solution can be obtained by the truncation method.

Note that constants x_n satisfying equations of system (36) determine an exact solution of the problem which takes into consideration all possible interactions of the shell and the spherical inclusion arising from multi-scattering. Components of expressions (36) and (37) have evident physical meaning. Free terms coincide with the coefficients defining radiation of the sound wave by a single sphere without accounting for the effects of multi-scattering. The sum describes the interaction between the bodies. It contains the multiplier $\mathrm{j}_n'(\omega r_0)/\mathrm{h}_n'(\omega r_0)$ which depends on the distance between interacting bodies. This multiplier decreases as the spherical inclusion dimensions (the wave distances ωr_0) decrease, and the role of interaction drops. Furthermore, there is the multiplier q_{mn}, which defines the "coefficients of sound diffraction" on the thin elastic cylindrical shell loaded with the ideal compressible fluid from inside and with the unbounded elastic medium from the outside. The function $M(\xi)$ which belongs to this multiplier defines the influence of the external elastic medium, and the function $R(\xi)$ defines the influence of the shell elastic properties. If these functions were equal to zero the multiplier q_{mn} would be represented by the function $\mathrm{H}_1(\sqrt{\omega^2-\xi^2})/\mathrm{J}_1(\sqrt{\omega^2-\xi^2})$ only and would express "coefficients of sound diffraction" on an absolutely rigid cylinder.

It should be noted that the above mathematical transformations admit the commutation of integrations and summations.

5. Numerical results

To find a numerical solution, the infinite system of algebraic Eq. (36) was truncated to a finite system of N equations. The order of truncation was determined experimentally by controlling the accuracy of satisfaction of the boundary conditions (about 10^{-4}).

All numerical results have been computed for the dimensionless variables (12). The compressible liquid parameters have been used as the normalization factors. The sound speed in the fluid inside the shell has been assumed to be equal $c_1 = 1500\,\mathrm{m/s}$ and its density $\gamma_1 = 10^3\,\mathrm{kg/m^3}$. The shell material has been considered to be such that the ratio of the internal liquid density to the density of this material is $f = 1/8$, $v = 0.3$ and $E = 2\times10^{11}\,\mathrm{N/m^2}$. For the medium external to the shell, the following parameters (corresponding to granite) have been chosen $c_s = 3000\,\mathrm{m/s}$, $\gamma_s = 3\times10^3\,\mathrm{kg/m^3}$, $E_s = 0.5\times10^{11}\,\mathrm{Pa}$, $v_s = 0.15$.

To calculate the integrals similar to integral (37), the interval of integration has been divided into the following intervals: $0 \leqslant \xi < \omega/a_l$, $\omega/a_l < \xi < \omega/a_t$, $\omega/a_t < \xi < \omega$, and $\omega < \xi < \infty$. The infinite limits of the integrals had to be truncated for numerical evaluation: they have been replaced by finite limits so as to provide convergence of the results obtained at least to the third decimal place. It should be noted that the subintegral functions have singular points inside the intervals of integration—these are variable values at which the subintegral functions increase without limit. These points are points at which the denominators of the integrands vanish, and the point $\xi = \omega$. These points have been placed into a sufficiently small ε-neighbourhood during the computations. An analysis of the subintegral functions behaviour in each ε-neighbourhood of the singular points has revealed that values of the subintegral functions have identical absolute magnitudes and opposite signs on the right and on the left of the singular points. Consequently, numerical results of the integration in the ε-neighbourhood of the singular points can be neglected.

The shell with wall thickness $h = 0.01$ has been considered in the numerical examples. Dimensionless values of the sphere radius have been varied within the limits of $r_0 = 0.25$ and 0.9. Two mechanical systems have been considered (depending on the character of distribution of the vibrating speed along the spherical inclusion surface): an infinite thin elastic cylindrical shell with ideal compressible liquid is submerged into an unbounded elastic medium and contains a spherical body which

(i) pulsates at the shell axis according to the law:

$$V(\theta) = 1, \tag{38}$$

(ii) oscillates along the axis of the shell according to the law:

$$V(\theta) = \cos(\theta). \tag{39}$$

Frequency dependencies of the hydrodynamic characteristics of the liquid inside the cylindrical volume have been investigated to find resonance phenomena in the systems considered. The influence of geometric dimensions of the spherical inclusion on these characteristics has also been studied. Comparison with the following resonance curves has been carried out: (i) with resonance curves of the plane axisymmetric hydroelastic system (which can be obtained from the considered spatial system by means of choosing the plane containing body centres ($z = 0$)), (ii) with resonance curves of the hydroelastic system which consist of the infinite thin elastic cylindrical shell with ideal compressible liquid, and also (iii) with resonance curves of a system consisting of an infinite thin elastic cylindrical shell with an ideal compressible liquid inside and an unbounded elastic medium.

Figs. 2–5 show results of the numerical investigation of the frequency characteristics of the systems considered in the diapason $0.2 \leqslant \omega \leqslant 10$. The graphic dependencies of absolute values of the hydrodynamic pressure of the fluid inside the elastic cylindrical shell and of the fluid particle speed versus the frequency of sphere vibrations inside the shell are shown on these figures.

Figs. 2 and 3 have been obtained for the case of pulsations of the spherical source on the shell axis according to law (38), and Figs. 4 and 5 and for the case of sphere oscillations along the shell axis in accordance with Eq. (39). Figs. 2 and 4 represent the frequency dependence of absolute values of the hydrodynamic pressure, and Figs. 3 and 5 the frequency dependence of the speed.

Different dimensions of the spherical source have been considered: Figs. 2(a), 3(a), 4(a), and 5(a), correspond to the sphere with radius $r_0 = 0.25$; Figs. 2(b), 3(b), 4(b), and 5(b) to the sphere with the radius $r_0 = 0.5$; Figs. 2(c), 3(c), 4(c), 5(c), and 2(d), 3(d), 4(d), 5(d) to the spheres with the radii $r_0 = 0.75$ and $r_0 = 0.9$, respectively. The mentioned characteristics of the pressure for both cases and of the speed for the case of the oscillating sphere (Figs. 2, 4, and 5) have been considered in the plane containing the cylinder axis ($\theta = 0$). The characteristics of the speed of liquid particles

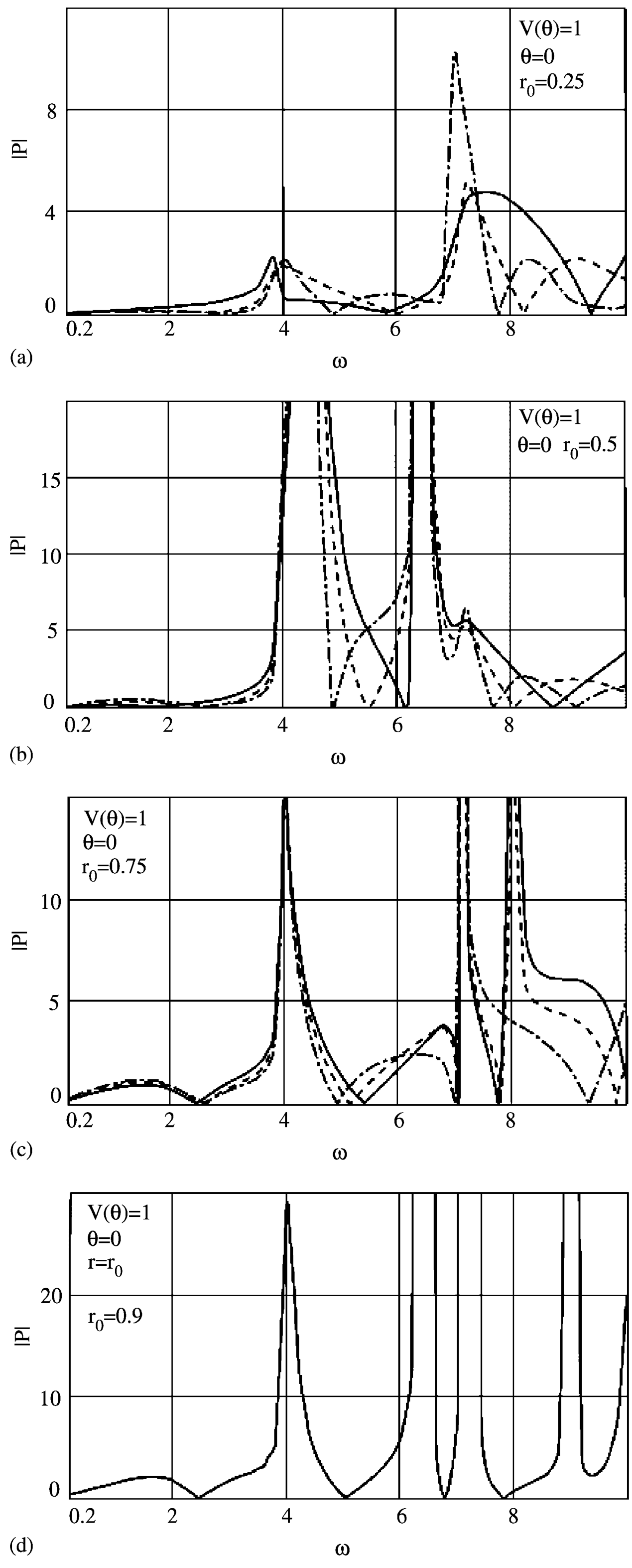

Fig. 2. Frequency characteristics of the hydrodynamic pressure inside the shell for the case of pulsations of spherical inclusions of different radii: —, on the surface of the sphere; - - -, at point $r = r_0 + (\rho_0 - r_0)/2$; $-\cdot-$, at $r = \rho_0$.

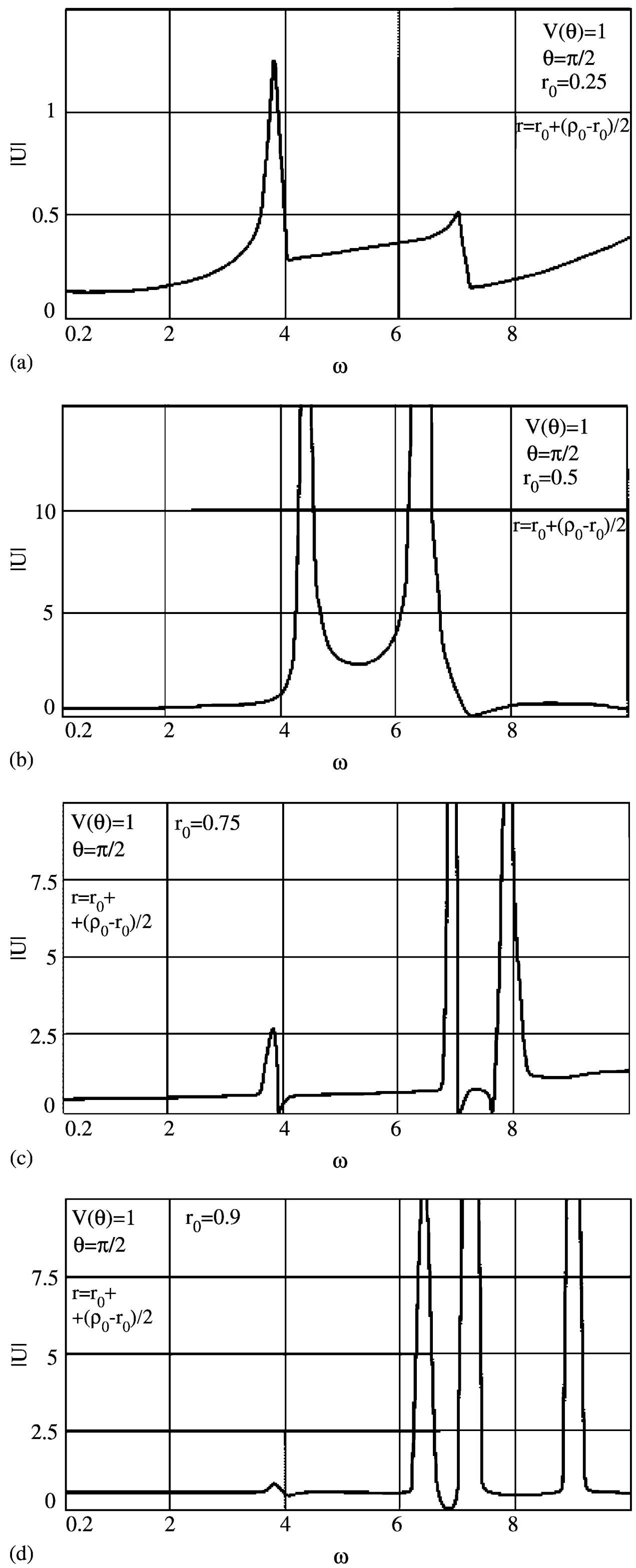

Fig. 3. Frequency characteristics of liquid speed inside the shell for the case of pulsations of spherical inclusions of different radii.

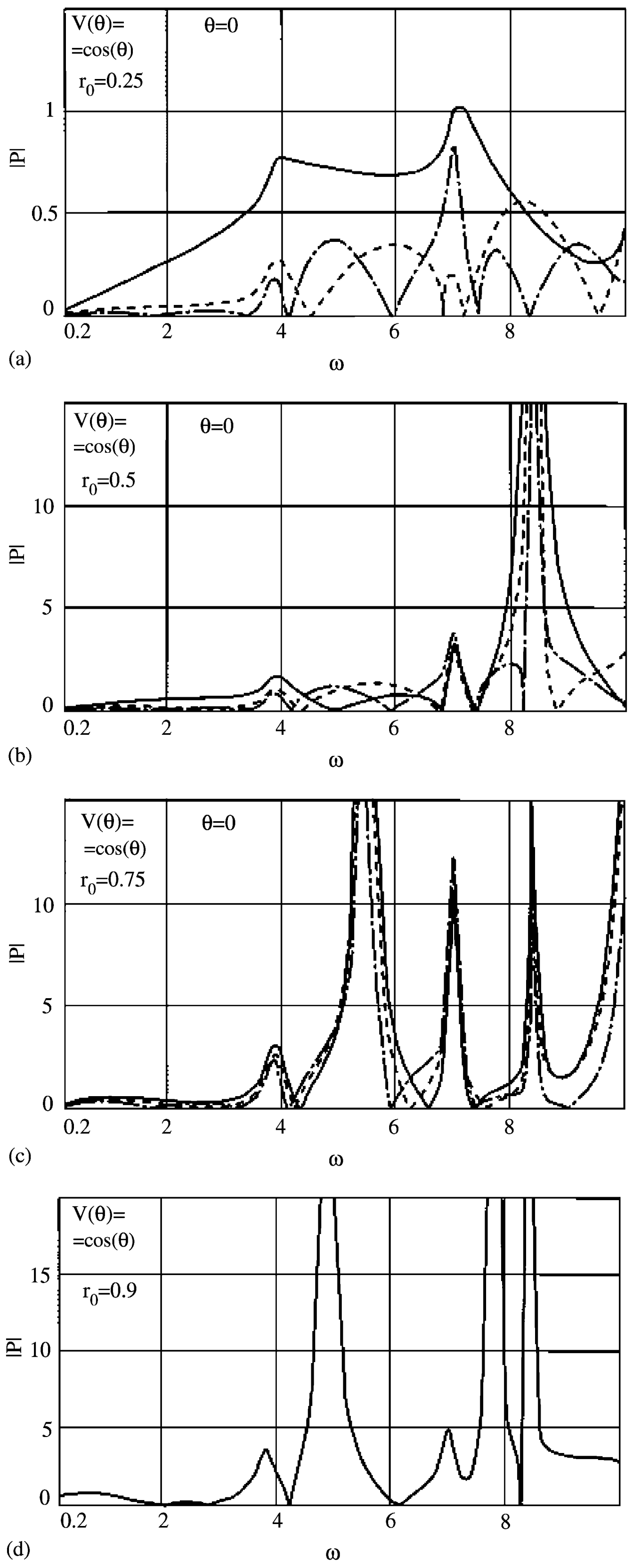

Fig. 4. Frequency characteristics of hydrodynamic pressure inside the shell for the case of oscillations of spherical inclusions of different radii; legend for different type of lines as in Fig. 2.

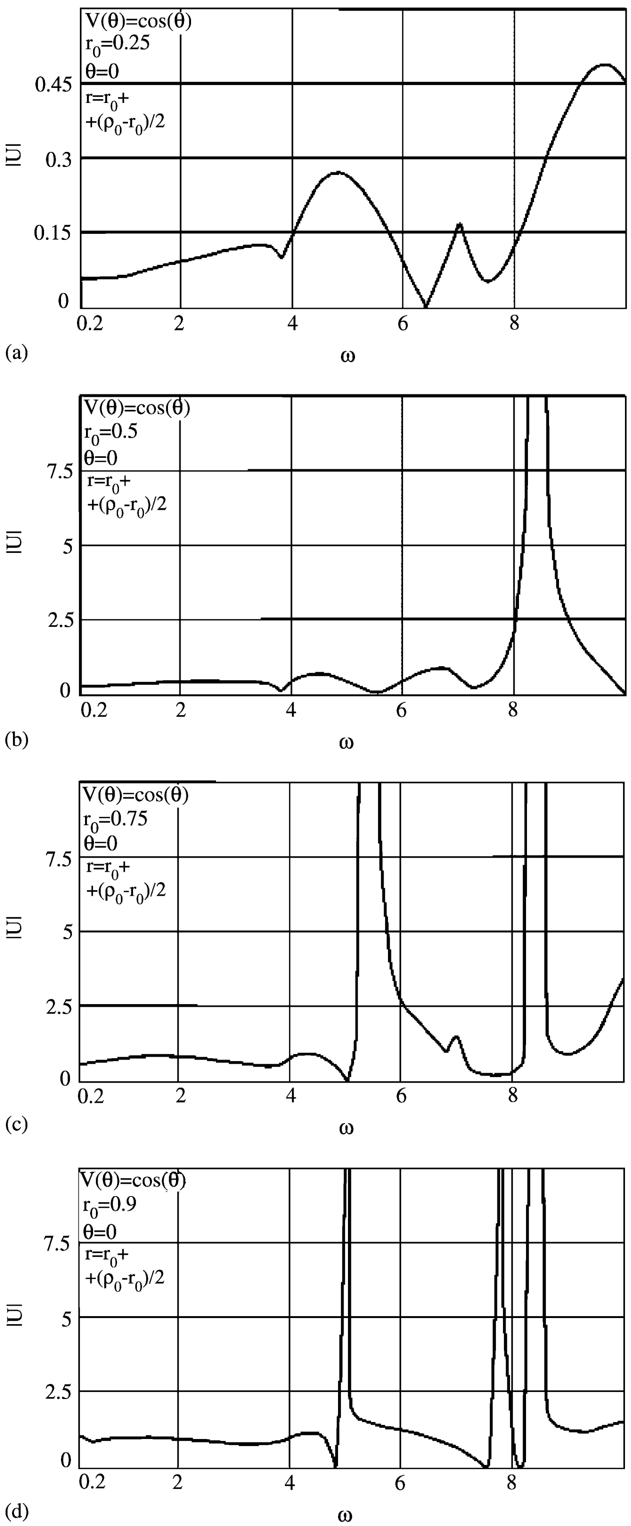

Fig. 5. Frequency characteristics of liquid speed inside the shell for the case of oscillations of spherical inclusions of different radii.

Table 1
Eigenfrequencies of the plane axis symmetric hydroelastic system for the interval $\omega \leqslant 20$

Sphere radius, r_0	ω_1	ω_2	ω_3	ω_4
0.25	4.448	8.537	12.679	16.842
0.5	6.393	12.625	18.889	
0.75	12.606			
0.9				

Table 2
Eigenfrequencies of the system without a spherical inclusion for the interval $\omega \leqslant 10$

System no.	ω_1	ω_2	ω_3	ω_4
1	2.572	5.282	8.151	11.099
2	3.832	7.016	10.173	

System no. 1: the infinite thin elastic cylindrical shell with ideal compressible liquid (the presence of the external medium has not been accounted for).
System no. 2: the infinite thin elastic cylindrical shell with ideal compressible liquid in the unbounded elastic medium.

for the case of the pulsating sphere (Fig. 2) have been considered in the plane $\theta = \pi/2$. The solid lines in Figs. 2 and 4 correspond to the pressure on the surface of the sphere ($r = r_0$); the dotted lines to the pressure calculated at the point $r = r_0 + (\rho_0 - r_0)/2$; the chain-dotted ones to the pressure at the point $r = \rho_0$. The frequency dependence of the absolute values of the speed have been calculated at the point $r = r_0 + (\rho_0 - r_0)/2$.

In the numerical example presented on Figs. 2(d) and 4(d) the curves calculated on the surface of the sphere; on the cylinder surface, and also in the space between them did not differ much from each other because of the sufficiently large sphere dimensions ($r_0 = 0.9$). Therefore, Figs. 2(d) and 4(d) represent only one of these characteristics: the pressure on the sphere surface.

Eigenfrequencies of the plane axissymmetric hydroelastic system from the frequency diapason $\omega \leqslant 10$ are given in Table 1 for comparison with the results of numerical investigation of ≪resonance≫ phenomena in the axisymmetric spatial system ≪the spherical inclusion—column of the ideal compressible liquid—the thin elastic infinite cylindrical shell—the unbounded elastic medium≫. Table 2 presents eigenfrequencies of the hydroelastic systems without a spherical inclusion: (i) of the system with an infinite thin elastic cylindrical shell filled with an ideal compressible fluid and (ii) of the system with an infinite thin elastic cylindrical shell filled with an ideal compressible fluid and submerged into an unbounded elastic medium.

6. Analysis of the results

By numerical investigation it has been established that there are "resonance" phenomena in the mechanical systems considered, since the systems have frequencies at which their characteristics (especially the hydrodynamic pressure and the speed of motion of the "internal" liquid particles) reach large amplitude values. It has also been established that such factors as presence and dimensions of the spherical inclusion have great influence on the values of the system eigenfrequencies.

It is interesting to note that, notwithstanding the spherical inclusion dimensions, both mechanical systems considered have frequencies at which the hydrodynamic pressure inside the shell reaches some finite increment of the amplitude. These maxima correspond to the resonances of the liquid column inside the shell. It is well known (Shenderov, 1972) that liquid in a vessel with absolutely rigid walls resounds at frequencies ω which can be determined as roots of the equation $J_1(\omega) = 0$ (first three values of these roots are 3.832; 7.016; 10.17).

As sphere dimensions grow, other maxima appear in the frequency characteristics of the systems investigated. These maxima correspond to "resonances" arising from the interaction of the system bodies: when superposition of waves many times scattered on these bodies leads to unlimited increment of the acoustic field amplitude. So, the number of eigenfrequencies of the axisymmetric system consisting of the spherical inclusion, column of the ideal compressible liquid the infinite thin elastic cylindrical shell, and the unbounded elastic medium in the interval $\omega \leqslant 10$ increases as the spherical inclusion radius increases (Figs. 2–5).

For example, for the system with the spherical inclusion with radius equal to a quarter of the shell radius (Figs. 2(a)–5(a)), none of the "eigenfrequencies" fall into the mentioned frequency interval; there are only maxima corresponding to the resonances of an infinite cylindrical liquid column: $\omega \approx 3.82$ and 7.02. Let us also note that these maxima correspond to the eigenfrequencies from the same frequency diapason of the system consisting of an infinite thin elastic cylindrical shell filled with an ideal compressible liquid and surrounded with an unbounded elastic medium (Table 2). This conclusion is confirmed by comparison of the results obtained and the results of calculations given by Kubenko and Dzyuba (2003) for the multi-coupled system consisting of the spherical inclusion, column of the ideal compressible liquid, and the infinite rigid cylinder.

It is interesting to note that eigenfrequencies of a shell with the same parameters (Table 2), which is not submerged in an external medium (such a problem statement does not take into consideration the presence of the external medium), are in the domain of lower frequencies. This can be explained by introducing some liquid mass on the internal side of the wall of the shell, which does not have external loading, vibrating jointly with the shell wall; whereas the wall of the shell loaded with an external medium can be considered as practically motionless.

The figures demonstrate that "resonances of interaction" depend on a manner of motion of the spherical inclusion inside the shell, i.e., they are not the same for each of the mechanical systems considered. Thus, in the case of the spherical body pulsations according to law (38) (Figs. 2 and 3), the values of the system eigenfrequencies corresponding to the "resonances of interaction" move into the domain of larger frequencies as the sphere radius increases. The eigenfrequencies of the plane axisymmetric system, obtained from the spatial one by extracting the plane containing the centres of bodies, have similar dependence as the circular cylindrical cavity dimensions increase (Table 1). But in the case of sphere oscillations according to law (39) (Figs. 4 and 5) the eigenfrequencies corresponding to the "resonances of interaction" move into the domain of lower frequency values as the sphere radius increases.

Such opposite behaviour can be explained as follows. In the mechanical system with the spherical body pulsating inside the cylindrical shell, vibrations are given by even modes, i.e., the total field of disturbances is represented by superposition of even vibration modes. The dominant mode in this case is the lowest vibration mode $n = 0$. When the sphere oscillates like a hard body along the shell axis, only odd modes contribute to the vibration process, and the lowest vibration mode with the number $n = 1$ is dominant.

7. Conclusions

An approach for solving the internal interaction problem for the system consisting of a thin elastic cylindrical shell submerged in an unbounded elastic medium, filled with an ideal compressible liquid and containing a spherical inclusion which vibrates in accordance with a predetermined law has been developed. The approach is based on applying addition theorems of the special functions and equations, which enable particular solutions of the Helmholtz equation in cylindrical coordinates to be constructed with the help of particular solutions of this equation in spherical coordinates, and vice versa. This enables the total solution to be written in the coordinate system of each body of the system, considered with the help of the superposition principle and taking into account the boundary conditions to be satisfied on their surfaces. As a result, the complex multi-coupled problem of interaction of bodies with different geometries is reduced to investigating and solving an infinite system of linear algebraic equations.

The approach suggested allows: (i) to get an exact analytical solution (as a Fourier series) of the interaction problem for a family of bodies with different geometries; (ii) to evaluate the speed and pressure fields of a compressible liquid and the deformation state of an external elastic medium and a cylindrical shell within an arbitrary precision; (iii) to study some applied and technological processes (such as vibrodisplacement and localization, purification and decontamination of liquid medium, airing and dispersion, technologies of resumption of oil production in foul wells) on the basis of more exact input data.

The theory has been tested numerically on a steel shell immersed in granite, filled with water and containing a vibrating spherical inclusion on its axis. The behaviour of the system has been investigated depending on the frequency of forced oscillations. The presence of "resonance" phenomena in such hydroelastic systems has been found by this study.

The methodology proposed in the present work has arisen during the realization of a joint project with the Sumy Department of oil-and-gas production devoted to modelling and investigating the problem of renewal of oil production in foul wells. Thus, it has already had actual practical application.

In conclusion, it should be noted that the method suggested can be extended to the case of an arbitrary number of spherical inclusions, and perhaps to the case of liquid flow inside the shell.

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Appendix A

The components of the vector of displacements of the elastic medium are given by

$$\begin{aligned} U_z &= \frac{\partial \Phi^{(2)}}{\partial z} + \frac{a_t}{\omega}\left(\frac{\omega^2}{a_t^2}\chi + \frac{\partial^2 \chi}{\partial z^2}\right), \\ U_\rho &= \frac{\partial \Phi^{(2)}}{\partial \rho} + \frac{1}{\rho}\frac{\partial \psi}{\partial \varphi} + \frac{a_t}{\omega}\frac{\partial^2 \chi}{\partial z \partial \rho}, \\ U_\varphi &= \frac{1}{\rho}\frac{\partial \Phi^{(2)}}{\partial \varphi} - \frac{\partial \psi}{\partial \rho} + \frac{a_t}{\omega}\frac{1}{\rho}\frac{\partial^2 \chi}{\partial z \partial \varphi}. \end{aligned} \tag{A.1}$$

The stresses in the elastic medium are given by

$$\begin{aligned} \sigma_{\rho\rho} &= \lambda\left(\frac{\partial U_z}{\partial z} + \frac{1}{\rho}\frac{\partial(\rho U_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial U_\varphi}{\partial \varphi}\right) + 2\mu\frac{\partial U_\rho}{\partial \rho}, \\ \sigma_{\rho z} &= \mu\left(\frac{\partial U_z}{\partial \rho} + \frac{\partial U_\rho}{\partial z}\right), \\ \sigma_{\rho\varphi} &= \mu\left[\rho\frac{\partial}{\partial \rho}\left(\frac{U_\varphi}{\rho}\right) + \frac{1}{\rho}\frac{\partial U_\rho}{\partial \varphi}\right]. \end{aligned} \tag{A.2}$$

Appendix B

The new symbols introduced in formula (23) are as follows:

$$\begin{aligned} b^{(2)}(\xi,\rho) &= \left((\lambda - 2\mu)\left(\frac{\omega^2}{a_l^2} - \xi^2\right) - \lambda\xi^2\right)\mathrm{H}_0\left(\sqrt{\frac{\omega^2}{a_l^2} - \xi^2}\rho\right) - 2(\lambda-\mu)\sqrt{\frac{\omega^2}{a_l^2} - \xi^2}\,\mathrm{H}_1\left(\sqrt{\frac{\omega^2}{a_l^2} - \xi^2}\rho\right)\Big/\rho, \\ d(\xi,\rho) &= 2(\lambda-\mu)\mathrm{i}\xi\frac{a_t}{\omega}\sqrt{\frac{\omega^2}{a_t^2} - \xi^2}\left[\sqrt{\frac{\omega^2}{a_t^2} - \xi^2}\,\mathrm{H}_0\left(\sqrt{\frac{\omega^2}{a_t^2} - \xi^2}\rho\right) - \mathrm{H}_1\left(\sqrt{\frac{\omega^2}{a_t^2} - \xi^2}\rho\right)\Big/\rho\right]. \end{aligned}$$

The symbols introduced in formulas (24) and (25) are given by

$$\begin{aligned} R(\xi) &= \frac{c^{*^2}(f/h)(\omega^2 c^{*^2} - \xi^2)}{v^2\xi^2 + (\omega^2 c^{*^2} - \xi^2)\left(1 + (h^2/12)\xi^4 - \omega^2 c^{*^2}\right)}, \\ R_u(\xi) &= \frac{\mathrm{i}v\xi}{\omega^2 c^{*^2} - \xi^2}R(\xi). \end{aligned}$$

Finally, the symbols introduced in formulas (32)–(34) are:

$$\begin{aligned} M(\xi) &= b^{(2)}(\xi,1)M^{(1)}(\xi) - d(\xi,1)M^{(1)}(\xi), \\ M^{(1)}(\xi) &= \left(R(\xi)\sqrt{\frac{\omega^2}{a_t^2} - \xi^2}\,\mathrm{H}_0\left(\sqrt{\frac{\omega^2}{a_t^2} - \xi^2}\right)\right. \\ &\left. -\mathrm{i}\xi R_u(\xi)\mathrm{H}_1\left(\sqrt{\frac{\omega^2}{a_t^2} - \xi^2}\right)\right)\Big/\mathrm{denom}(\xi), \end{aligned}$$

$$M^{(2)}(\xi) = \frac{\left[R_u(\xi)\sqrt{(\omega^2/a_l^2) - \xi^2}H_1\left(\sqrt{(\omega^2/a_l^2) - \xi^2}\right) + \mathrm{i}\xi R(\xi)\mathrm{H}_0\left(\sqrt{(\omega^2/a_l^2) - \xi^2}\right)\right]}{\left[(a_t/\omega)\sqrt{(\omega^2/a_t^2) - \xi^2}\mathrm{denom}(\xi)\right]},$$

$$\mathrm{denom}(\xi) = \sqrt{\frac{\omega^2}{a_t^2} - \xi^2}\mathrm{H}_0\left(\sqrt{\frac{\omega^2}{a_t^2} - \xi^2}\right)\sqrt{\frac{\omega^2}{a_l^2} - \xi^2}\,\mathrm{H}_1\left(\sqrt{\frac{\omega^2}{a_l^2} - \xi^2}\right) - \xi^2\mathrm{H}_1\left(\sqrt{\frac{\omega^2}{a_t^2} - \xi^2}\right)\mathrm{H}_0\left(\sqrt{\omega^2/a_l^2 - \xi^2}\right).$$

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